Description of fluid! – (what is a fluid?)

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This text is a short version of Sections 2.1, 2.2 and 2.4 (pp 11-16, 20) of my PhD thesis published in 2003 and concern the explanation of why large particle systems (with dry or wet particles), like fresh concrete, rings of Saturn or sea ice floes, can be treated as a fluid with the same fundamental properties as for example water. I wrote this part of my thesis in around 2001. The PhD thesis is as follows:

J.E. Wallevik, Rheology of Particle Suspensions - Fresh Concrete, Mortar and Cement Paste with Various Types of Lignosulfonates, Ph.D. thesis, Department of Structural Engineering, The Norwegian University of Science and Technology, Trondheim, 2003.

If the following text is used, please make a reference to the above mentioned theses. This thesis can be download from:

https://brage.bibsys.no/xmlui/handle/11250/236410

(you can also search for "jon wallevik" in brage.bibsys.no and you should find the thesis)

What is a fluid? When considering a large collection of rock and ice fragments, as in the rings of Saturn, or a large collection of sea ice floes in the Icelandic waters, one might have difficulties in accepting those two systems as fluids. Of course, in their isolated state, a single rock or ice fragment, or a single sea ice floe, does not represent such state. But with a large collection of these solid particles, those two separated systems can be presented as two different types of fluid. They are classified and characterized by their potential solid ice/rock and sea ice interactions through parameters called viscosity.

The governing equation represents the primary law used when simulating the flow of the above mentioned two fluids. Its philosophy is the Newton's second law "$m\, d\mathbf{v}/dt = \mathbf{F}$". For Newtonian fluid, the governing equation is often designated as **Navier-Stokes**[1]. However, for non-Newtonian fluids, the governing equation is more complicated and is then named the **Cauchy equation of motion**, given by [1,2]

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g} \ . \qquad (1)$$

The term $\mathbf{v}$ is the fluid velocity, ρ is the density, t is the time, $\mathbf{g}$ is the gravity and $\boldsymbol{\sigma}$ is the (total) stress tensor [1,2]. The stress tensor is given by

$$\boldsymbol{\sigma} = -p\,\mathbf{I} + \mathbf{T} \quad . \qquad (2)$$

In general and as shown above, the stress tensor is decomposed into an isotropic pressure term p and an extra stress contribution $\mathbf{T}$, commonly known as the extra stress tensor [3]. The term $\mathbf{I}$ is the unit dyadic.

[1] With both the shear viscosity η (i.e. apparent viscosity) and the bulk viscosity κ present (and both values are constants), the equation is referred to as Navier–Stokes–Duhem [1].

In fundamental terms, *Fluid Mechanics* consist of solving Eq. (1). Although the term "*single fluid approach*" or "single fluid technique" are frequently used when this is done, the reader should not misunderstand that the fluid must be smooth, liquid like, continuous, consisting of fine particles only, or anything like that.

To demonstrate versatility, then in [4] (in the beginning of Chapter 2) the *Cauchy equation of motion*, Eq. (1), is derived by using Newton's second law for each and every solid particle that resides within a fluid particle. As shown in Fig. 1, a single **fluid particle**[2] is composed of large number of **solid particles**. The velocity of the fluid particle is the mass average velocity of all the solid particles composing the specific fluid particle [4]. Hence, any random and spontaneous velocity contributions from the individual solid particles are summarised out in the averaging and only the relevant smooth motion of the flow remains [4]. In this relation, water molecules and other similar sized particles are also considered as solid particles, just as sand or aggregate particles. The only physical requirement during the above-mentioned derivation is that all particle motion behaviour and all particle-particle interactions (of any kind, e.g. van der Waals, hard sphere collisions and so forth) comply with the Newton's second and third law, i.e. of motion and of action and reaction. The extra stress tensor **T** then describes the fluid particle resistance to deform. This resistance is made by the rate of momentum transfer between particles during the (rate of) deformation (see [5], as well as [4] in Section 2.4: "*Solid Particle Interaction and Viscosity*").

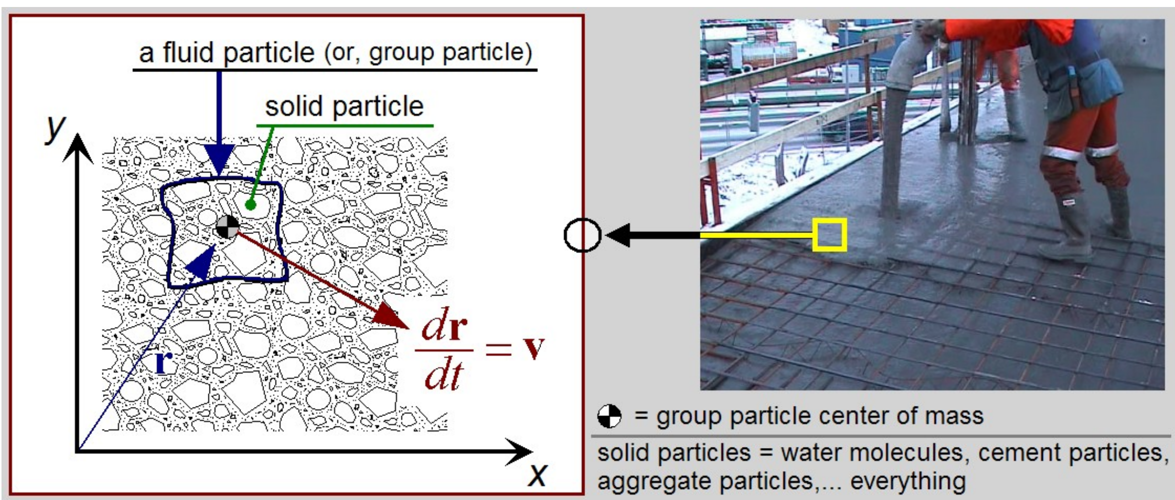

Fig. 1: The difference between solid particle and fluid particle (or group particle).

The choice of boundary (i.e. dimensions) of a fluid particle, shown for example with roughly square like geometry in Fig 1 (the geometrical form is arbitrary chosen in this figure) is determined by the number of solid particles that it is composed of. This number is again determined by the requirement of attaining smooth velocity[3] for the fluid particle, as described in the previous paragraph. In the end, this choice becomes more or less hidden when utilizing Eq. (1). It should be clear that this concept is better

[2] The term ***Fluid Particle*** is for example used in [6], while the designation ***Continuum Particle*** is used in [4] (acronym CP) for the same particle. Shortly, this particle will also be referred to as ***Group Particle***. Other names for this type of particle are ***Material Point*** [6,7,8] and ***Fluid Parcel*** [9]. Without doubt, there exist more designations for this particular phenomenon.

[3] As random motion (Brownian motion) of water molecules (in water) is not understood as fluid mechanical turbulence, then neither are the random and spontaneous velocity contributions of individual solid particles (composing the fluid particle) understood as such. Rather, when groups of fluid particles start to travel coherently in circular paths, as a part of eddies or vortices of varying size, it is possible to define turbulence. More precisely, turbulent flow is characterized by a mixing action caused by eddies of varying size, throughout the flow continuum [10].

explained with mathematics as is done in [4]. Unfortunately, that approach cannot be repeated here as such would take too much space for the current introduction text.

From the above text, it should be evident that the solid particles (composing the fluid particle) can be of any type, shape and size. Thus when using Eq. (1), the subject "fluid" doesn't even have to consist of liquid or similar "smooth" materials or of the same material (e.g. of water molecules only as in pure water), at all. This is demonstrated in [4] (Appendix B.5) where the velocity profile of the rings of Saturn, is accurately calculated by using Eq. (1). The ring consists of solid ice and rock fragments ("dry materials"), with dimensions ranging from few centimetres to few meters across. This calculation is done to highlight that the "*single fluid approach*" is in essence the theory of collective motion of a very large number of solid particles. The largest solid particles are treated in the exactly the same manner as the smallest ones. Thus, the term "*group calculation*", "*group fluid calculation*" or "*group approach*" might more appropriate than the misleading term "*single fluid approach*". Even the term "*fluid*" is also misleading in relation to the use of Eq. (1), as this equation can be used on "*non-fluid*" (or *non-liquid*) materials as for example the rings of Saturn. Thus, the term "*group particle*" (as in *group of solid particles*) is more appropriate than "*fluid particle*".

There are lot of mathematical and physical issues about the group particle (or fluid particle) that are not dealt with here. Such discussion would simply take too much time and space for the current section. However, for the interested reader, the whole concept is very well explained in [4], then in Chapter 2 as well as in Appendix B. For example, since the group particle consists of finite dimensions and is not a point item, then in Appendix B.4, topics like the *Resolution of the Material Space* is addressed in relation to Eq. (1). There it is explained that the name *continuity equation* is given because this equation assumes that the velocity and density are defined in every point in space [6]. This assumption is in general valid for a coarse particle suspension, like the rings of Saturn, the fresh concrete and so forth.

Summary: *We can recreate the Cauchy equation (and thus one of its offspring like the Navier-Stokes) by looking at the Newton's second and third law for every solid particle within a fluid particle. But we cannot resolve (i.e. calculate) the motion inside the fluid particle (this can be done for example with the discrete element method (DEM), but is very time consuming). Both the continuous- and the discrete approach (DEM) own its basis in Newton's second- and third law for every particle of the system. The former method does so in an implicit manner, while the latter does so explicitly. For the Cauchy equation (and its offspring, like the Navier-Stokes), the effect of the average particle – particle interactions inside a fluid particle is bundled into a parameter called viscosity (i.e. shear viscosity, bulk viscosity etc.). For the discrete approach this is done directly by calculating contact forces between particles.*

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